



WORKSHEET

Combination and Permutation Question

Question Description: This style of question on permutations and combinations and they ask a question on this every year. Simply put they ask you to use your understanding of permutations and combinations to make different 'selections' or arrangements of objects with various limitations.

$$C(n, r) = \frac{n!}{r!(n - r)!}$$

$$P(n, r) = \frac{n!}{(n - r)!}$$

Choosing 'r' things from a group of 'n' things

"ABC" Example

6 Permutations (Each arrangement treated as different and counted "order matters")

"ABC, ACB, BAC, BCA, CAB, CBA"

1 Combination (Each arrangement is counted as the same "order doesn't matter")

"ABC = ACB = BAC = BCA = CAB = CBA"

QUESTIONS

Q1 s2015_p61

- (a) Find how many different numbers can be made by arranging all nine digits of the number 223 677 888 if
- (i) there are no restrictions, [2]
 - (ii) the number made is an even number. [4]
- (b) Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

Q2 s2015_p62

- (a) Find the number of different ways the 7 letters of the word BANANAS can be arranged
- (i) if the first letter is N and the last letter is B, [3]
 - (ii) if all the letters A are next to each other. [3]
- (b) Find the number of ways of selecting a group of 9 people from 14 if two particular people cannot both be in the group together. [3]

Q3 s2015_p63

Rachel has 3 types of ornament. She has 6 different wooden animals, 4 different sea-shells and 3 different pottery ducks.

- (i) She lets her daughter Cherry choose 5 ornaments to play with. Cherry chooses at least 1 of each type of ornament. How many different selections can Cherry make? [5]

Rachel displays 10 of the 13 ornaments in a row on her window-sill. Find the number of different arrangements that are possible if

- (ii) she has a duck at each end of the row and no ducks anywhere else, [3]
- (iii) she has a duck at each end of the row and wooden animals and sea-shells are placed alternately in the positions in between. [3]

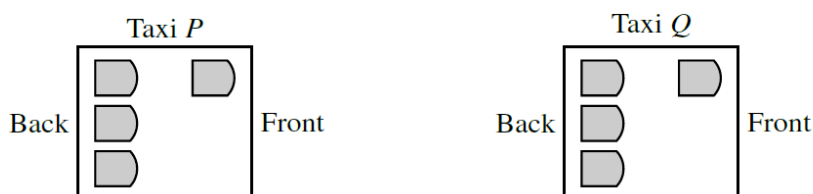
Q4 w2015_p61

- (a) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged
- (i) if all the letters E are together, [3]
 - (ii) if the T is at one end and there is an S at the other end. [3]
- (b) Four letters are selected from the nine letters of the word VENEZUELA. Find the number of possible selections which contain exactly one E. [3]

Q5 w2015_p62

A group of 8 friends travels to the airport in two taxis, P and Q . Each taxi can take 4 passengers.

- (i) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q , with Jon and Sarah travelling in the same taxi. Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

- (ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

Q6 w2015_p63

- (a) Find the number of different ways that the 13 letters of the word ACCOMMODATION can be arranged in a line if all the vowels (A, I, O) are next to each other. [3]
- (b) There are 7 Chinese, 6 European and 4 American students at an international conference. Four of the students are to be chosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one Chinese and at least one European student are included. [5]

Q7 s2016_p61

- (a) (i) Find how many numbers there are between 100 and 999 in which all three digits are different. [3]
- (ii) Find how many of the numbers in part (i) are odd numbers greater than 700. [4]
- (b) A bunch of flowers consists of a mixture of roses, tulips and daffodils. Tom orders a bunch of 7 flowers from a shop to give to a friend. There must be at least 2 of each type of flower. The shop has 6 roses, 5 tulips and 4 daffodils, all different from each other. Find the number of different bunches of flowers that are possible. [4]

Q8 s2016_p62

- (a) Find the number of different arrangements which can be made of all 10 letters of the word WALLFLOWER if
- (i) there are no restrictions, [1]
 - (ii) there are exactly six letters between the two Ws. [4]
- (b) A team of 6 people is to be chosen from 5 swimmers, 7 athletes and 4 cyclists. There must be at least 1 from each activity and there must be more athletes than cyclists. Find the number of different ways in which the team can be chosen. [4]

Q9 s2016_p63

Find the number of ways all 9 letters of the word EVERGREEN can be arranged if

- (i) there are no restrictions, [1]
- (ii) the first letter is R and the last letter is G, [2]
- (iii) the Es are all together. [2]

Three letters from the 9 letters of the word EVERGREEN are selected.

- (iv) Find the number of selections which contain no Es and exactly 1 R. [1]
- (v) Find the number of selections which contain no Es. [3]

Q10 w2016_p61

- (a) Find the number of different ways of arranging all nine letters of the word PINEAPPLE if no vowel (A, E, I) is next to another vowel. [4]
- (b) A certain country has a cricket squad of 16 people, consisting of 7 batsmen, 5 bowlers, 2 all-rounders and 2 wicket-keepers. The manager chooses a team of 11 players consisting of 5 batsmen, 4 bowlers, 1 all-rounder and 1 wicket-keeper.
- (i) Find the number of different teams the manager can choose. [2]
 - (ii) Find the number of different teams the manager can choose if one particular batsman refuses to be in the team when one particular bowler is in the team. [3]

Q11 w2016_p62

Find the number of ways all 10 letters of the word COPENHAGEN can be arranged so that

- (i) the vowels (A, E, O) are together and the consonants (C, G, H, N, P) are together, [3]
- (ii) the Es are not next to each other. [4]

Four letters are selected from the 10 letters of the word COPENHAGEN.

- (iii) Find the number of different selections if the four letters must contain the same number of Es and Ns with at least one of each. [5]

Q12 w2016_p63

Numbers are formed using some or all of the digits 4, 5, 6, 7 with no digit being used more than once.

- (i) Show that, using exactly 3 of the digits, there are 12 different odd numbers that can be formed. [3]
- (ii) Find how many odd numbers altogether can be formed. [3]

Q13 s2017_p61

- (a) Eight children of different ages stand in a random order in a line. Find the number of different ways this can be done if none of the three youngest children stand next to each other. [3]
- (b) David chooses 5 chocolates from 6 different dark chocolates, 4 different white chocolates and 1 milk chocolate. He must choose at least one of each type. Find the number of different selections he can make. [4]
- (c) A password for Chelsea's computer consists of 4 characters in a particular order. The characters are chosen from the following.
 - The 26 capital letters A to Z
 - The 9 digits 1 to 9
 - The 5 symbols # ~ * ? !

The password must include at least one capital letter, at least one digit and at least one symbol. No character can be repeated. Find the number of different passwords that Chelsea can make. [4]

Q14 s2017_p62

A library contains 4 identical copies of book *A*, 2 identical copies of book *B* and 5 identical copies of book *C*. These 11 books are arranged on a shelf in the library.

- (i) Calculate the number of different arrangements if the end books are either both book *A* or both book *B*. [4]
- (ii) Calculate the number of different arrangements if all the books *A* are next to each other and none of the books *B* are next to each other. [5]

Q15 s2017_p63

- (a) Find how many numbers between 3000 and 5000 can be formed from the digits 1, 2, 3, 4 and 5,
- (i) if digits are not repeated, [2]
 - (ii) if digits can be repeated and the number formed is odd. [3]
- (b) A box of 20 biscuits contains 4 different chocolate biscuits, 2 different oatmeal biscuits and 14 different ginger biscuits. 6 biscuits are selected from the box at random.
- (i) Find the number of different selections that include the 2 oatmeal biscuits. [2]
 - (ii) Find the probability that fewer than 3 chocolate biscuits are selected. [4]

Q16 w2017_p61

- (a) A village hall has seats for 40 people, consisting of 8 rows with 5 seats in each row. Mary, Ahmad, Wayne, Elsie and John are the first to arrive in the village hall and no seats are taken before they arrive.
- (i) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John assuming there are no restrictions? [2]
 - (ii) How many possible arrangements are there of seating Mary, Ahmad, Wayne, Elsie and John if Mary and Ahmad sit together in the front row and the other three sit together in one of the other rows? [4]
- (b) In how many ways can a team of 4 people be chosen from 10 people if 2 of the people, Ross and Lionel, refuse to be in the team together? [4]

Q17 w2017_p62

- (a) Find the number of different 3-digit numbers greater than 300 that can be made from the digits 1, 2, 3, 4, 6, 8 if
- (i) no digit can be repeated, [3]
 - (ii) a digit can be repeated and the number made is even. [3]
- (b) A team of 5 is chosen from 6 boys and 4 girls. Find the number of ways the team can be chosen if
- (i) there are no restrictions, [1]
 - (ii) the team contains more boys than girls. [3]

Q18 w2017_p63

A car park has spaces for 18 cars, arranged in a line. On one day there are 5 cars, of different makes, parked in randomly chosen positions and 13 empty spaces.

- (i) Find the number of possible arrangements of the 5 cars in the car park. [2]
- (ii) Find the probability that the 5 cars are not all next to each other. [5]

On another day, 12 cars of different makes are parked in the car park. 5 of these cars are red, 4 are white and 3 are black. Elizabeth selects 3 of these cars.

- (iii) Find the number of selections Elizabeth can make that include cars of at least 2 different colours. [5]

Q19 s2018_p61

Find the number of different ways in which all 9 letters of the word MINCEMEAT can be arranged in each of the following cases.

- (i) There are no restrictions. [1]
- (ii) No vowel (A, E, I are vowels) is next to another vowel. [4]

5 of the 9 letters of the word MINCEMEAT are selected.

- (iii) Find the number of possible selections which contain exactly 1 M and exactly 1 E. [2]
- (iv) Find the number of possible selections which contain at least 1 M and at least 1 E. [3]

Q20 s2018_p62

- (a) Find the number of ways in which all 9 letters of the word AUSTRALIA can be arranged in each of the following cases.

- (i) All the vowels (A, I, U are vowels) are together. [3]
 - (ii) The letter T is in the central position and each end position is occupied by one of the other consonants (R, S, L). [3]
- (b) Donna has 2 necklaces, 8 rings and 4 bracelets, all different. She chooses 4 pieces of jewellery. How many possible selections can she make if she chooses at least 1 necklace and at least 1 bracelet? [4]

Q21 s2018_p63

Find the number of ways the 9 letters of the word SEVENTEEN can be arranged in each of the following cases.

- (i) One of the letter Es is in the centre with 4 letters on either side. [2]
 - (ii) No E is next to another E. [3]
- 5 letters are chosen from the 9 letters of the word SEVENTEEN.
- (iii) Find the number of possible selections which contain exactly 2 Es and exactly 2 Ns. [1]
 - (iv) Find the number of possible selections which contain at least 2 Es. [4]

Q22 w2018_p61

In an orchestra, there are 11 violinists, 5 cellists and 4 double bass players. A small group of 6 musicians is to be selected from these 20.

- (i) How many different selections of 6 musicians can be made if there must be at least 4 violinists, at least 1 cellist and no more than 1 double bass player? [4]

The small group that is selected contains 4 violinists, 1 cellist and 1 double bass player. They sit in a line to perform a concert.

- (ii) How many different arrangements are there of these 6 musicians if the violinists must sit together? [3]

Q23 w2018_p62

- (i) How many different arrangements are there of the 11 letters in the word MISSISSIPPI? [2]
- (ii) Two letters are chosen at random from the 11 letters in the word MISSISSIPPI. Find the probability that these two letters are the same. [3]

Q24 s2019_p61

Freddie has 6 toy cars and 3 toy buses, all different. He chooses 4 toys to take on holiday with him.

- (i) In how many different ways can Freddie choose 4 toys? [1]
- (ii) How many of these choices will include both his favourite car and his favourite bus? [2]

Freddie arranges these 9 toys in a line.

- (iii) Find the number of possible arrangements if the buses are all next to each other. [3]
- (iv) Find the number of possible arrangements if there is a car at each end of the line and no buses are next to each other. [3]

Q25 s2019_p62

- (a) A group of 6 teenagers go boating. There are three boats available. One boat has room for 3 people, one has room for 2 people and one has room for 1 person. Find the number of different ways the group of 6 teenagers can be divided between the three boats. [3]
- (b) Find the number of different 7-digit numbers which can be formed from the seven digits 2, 2, 3, 7, 7, 7, 8 in each of the following cases.
- (i) The odd digits are together and the even digits are together. [3]
- (ii) The 2s are not together. [4]

Q26 s2019_p63

Mr and Mrs Keene and their 5 children all go to watch a football match, together with their friends Mr and Mrs Uzuma and their 2 children. Find the number of ways in which all 11 people can line up at the entrance in each of the following cases.

- (i) Mr Keene stands at one end of the line and Mr Uzuma stands at the other end. [2]
- (ii) The 5 Keene children all stand together and the Uzuma children both stand together. [3]

Q27 w2019_p61

- (i) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that all four Es are together. [1]
- (ii) Find the number of different ways in which all 12 letters of the word STEEPLECHASE can be arranged so that the Ss are not next to each other. [4]

Four letters are selected from the 12 letters of the word STEEPLECHASE.

- (iii) Find the number of different selections if the four letters include exactly one S. [4]

Q28 w2019_p62

- (i) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that all three Os are together and both Ts are together. [1]
- (ii) Find the number of different ways in which the 9 letters of the word TOADSTOOL can be arranged so that the Ts are not together. [4]
- (iii) Find the probability that a randomly chosen arrangement of the 9 letters of the word TOADSTOOL has a T at the beginning and a T at the end. [2]
- (iv) Five letters are selected from the 9 letters of the word TOADSTOOL. Find the number of different selections if the five letters include at least 2 Os and at least 1 T. [4]

Q29 w2019_p63

- (i) How many different arrangements are there of the 9 letters in the word CORRIDORS? [2]
- (ii) How many different arrangements are there of the 9 letters in the word CORRIDORS in which the first letter is D and the last letter is R or O? [3]

Q30 w2019_p63

A sports team of 7 people is to be chosen from 6 attackers, 5 defenders and 4 midfielders. The team must include at least 3 attackers, at least 2 defenders and at least 1 midfielder.

- (i) In how many different ways can the team of 7 people be chosen? [4]

The team of 7 that is chosen travels to a match in two cars. A group of 4 travel in one car and a group of 3 travel in the other car.

- (ii) In how many different ways can the team of 7 be divided into a group of 4 and a group of 3? [2]

ANSWERS

Q1

(a) (i)	$\frac{9!}{2!2!3!}$ $= 15120 \text{ ways}$	B1 B1 [2]	Dividing by 2!2!3! Correct answer																		
(ii)	*****3 in $\frac{8!}{2!2!3!} = 1680 \text{ ways}$ *****7 in $\frac{8!}{2!3!} = 3360 \text{ ways}$ Total even = 15120 – 1680 – 3360 = 10080 ways OR *****2 in $\frac{8!}{2!3!} = 3360 \text{ ways}$ *****6 in $\frac{8!}{2!2!3!} = 1680 \text{ ways}$ *****8 in $\frac{8!}{2!2!2!} = 5040 \text{ways}$ Total = 10080 ways OR “15120” $\times 6/9 = 10080$	B1 B1 M1 A1 [4] B1 B1 M1 A1 M2 A2	Correct ways end in 3 Correct ways end in 7 Finding odd and subtr from 15120 or their (i) Correct answer One correct way end in even correct way end in another even Summing 2 or 3 ways Correct answer Mult their (i) by 2/3 oe Correct answer																		
(b)	T(3) S(6) G(14) <table><tr><td>1</td><td>1</td><td>3 in $3 \times 6 \times {}^{14}C_3 = 6552$</td></tr><tr><td>1</td><td>3</td><td>1 in $3 \times {}^6C_3 \times 14 = 840$</td></tr><tr><td>3</td><td>1</td><td>1 in $1 \times 6 \times 14 = 84$</td></tr><tr><td>2</td><td>2</td><td>1 in ${}^3C_2 \times {}^6C_2 \times 14 = 630$</td></tr><tr><td>2</td><td>1</td><td>2 in ${}^3C_2 \times 6 \times {}^{14}C_2 = 1638$</td></tr><tr><td>1</td><td>2</td><td>2 in $3 \times {}^6C_2 \times {}^{14}C_2 = 4095$</td></tr></table> Total ways = 13839 (13800)	1	1	3 in $3 \times 6 \times {}^{14}C_3 = 6552$	1	3	1 in $3 \times {}^6C_3 \times 14 = 840$	3	1	1 in $1 \times 6 \times 14 = 84$	2	2	1 in ${}^3C_2 \times {}^6C_2 \times 14 = 630$	2	1	2 in ${}^3C_2 \times 6 \times {}^{14}C_2 = 1638$	1	2	2 in $3 \times {}^6C_2 \times {}^{14}C_2 = 4095$	M1 M1 M1 B1 A1 [5]	Mult 3 (combinations) together assume $6 = {}^6C_1$ etc Listing at least 4 different options Summing at least 4 different options At least 3 correct numerical options Correct answer
1	1	3 in $3 \times 6 \times {}^{14}C_3 = 6552$																			
1	3	1 in $3 \times {}^6C_3 \times 14 = 840$																			
3	1	1 in $1 \times 6 \times 14 = 84$																			
2	2	1 in ${}^3C_2 \times {}^6C_2 \times 14 = 630$																			
2	1	2 in ${}^3C_2 \times 6 \times {}^{14}C_2 = 1638$																			
1	2	2 in $3 \times {}^6C_2 \times {}^{14}C_2 = 4095$																			

Q2

(a) (i)	N*****B Number of ways = $\frac{5!}{3!}$ = 20	B1 B1 B1 3	5! Seen in num oe or alone mult by $k \geq 1$ 3! Seen in denom can be mult by $k \geq 1$ Correct final answer
(ii)	B(AAA)NNS Number of ways = $\frac{5!}{2!}$ or 5P_3 = 60	M1 M1 A1 3	5! seen as a num can be mult by $k \geq 1$ Dividing by 2! Correct final answer
(b)	${}^{14}C_9$ total options = 2002 T and M both in ${}^{12}C_7 = 792$ Ans $2002 - 792 = 1210$ OR Neither in ${}^{12}C_9 = 220$ One in ${}^{12}C_8 = 495$ Other in ${}^{12}C_8 = 495$	M1 B1 A1 3 M1 B1	${}^{14}C_9$ or ${}^{14}P_9$ in subtraction attempt ${}^{12}C_7$ (792) seen Correct final answer Summing 2 or 3 options at least 1 correct condone ${}^{12}P_9 + {}^{12}P_8 + {}^{12}P_8$ here only Second correct option seen accept another 495 or if M1 not awarded, any correct option
	total = 1210	A1	Correct final answer

Q3

(i)	W S D 1 1 3 = $6 \times 4 \times {}^3C_3 = 24$ 1 3 1 = $6 \times {}^4C_3 \times 3 = 72$ 3 1 1 = ${}^6C_3 \times 4 \times 3 = 240$ 1 2 2 = $6 \times {}^4C_2 \times {}^3C_2 = 108$ 2 1 2 = ${}^6C_2 \times 4 \times {}^3C_2 = 180$ 2 2 1 = ${}^6C_2 \times {}^4C_2 \times 3 = 270$ Total = 894	M1 M1 M1 B1 A1 [5]	Listing at least 4 different options Mult 3 (combs) together assume $6 = {}^6C_1, \Sigma r=5$ Summing at least 4 different evaluated/unsimplified options >1 At least 3 correct unsimplified options Correct answer
(ii)	${}^3P_2 \times {}^{10}P_8$ = 10886400	B1 B1 B1 [3]	3P_2 oe seen multiplied either here or in (iii) $k^{10}P_x$ seen or k^yP_8 with no addition, $k \geq 1, y > 8, x < 10$ Correct answer, nfw
(iii)	DSWSWSWSWD or DWSWSWSWSWD D in 3P_2 ways = 6 S in 4P_4 ways = 24 W in ${}^6P_4 = 360$ Swap SW in 2 ways Total = 103680 ways	B1 B1 B1 [3]	If 3P_2 has not gained credit in (ii) may be awarded 4P_4 or 6P_4 oe seen multiplied or common in all terms (no division) Mult by 2 (condone 2!) Correct answer, 3sf or better, nfw

Q4

(i)	5 (i) eg **(EEEE)*** Number of ways = $\frac{6!}{2!2!} = 180$	M1 M1 A1 [3]	Mult by 6! oe Dividing by 2!2! oe Correct answer
(ii)	S*****T or T*****S Number of ways = $\frac{7!}{4!2!} \times 2$ = 210	M1 M1 A1 [3]	Mult by 7! Or dividing by one of 2! or 4! Mult by 2 Correct answer
(iii)	exactly one E in 6C_3 ways = 20	M1 M1 A1 [3]	6C_x as a single answer ${}_xC_3$ as a single answer correct answer

Q5

(i)	Two in same taxi: ${}^6C_2 \times {}^4C_4 \times 2$ or ${}^6C_2 + {}^6C_4$ = 30	M1 M1 A1 3	6C_4 or 6C_2 oe seen anywhere 'something' $\times 2$ only or adding 2 equal terms Correct final answer
(ii)	MJS in taxi $({}^5C_1 \times 2 \times 2) \times {}^4P_4$ = 480	M1 M1 M1 A1 4	5P_1 , 5C_1 or 5 seen anywhere Mult by 2 or 4 oe Mult by 4P_4 oe eg 4! or $4 \times {}^3P_3$ or can be part of 5! Correct final answer

Q6

(a)	e.g. **(AAOOOI)***** $\frac{8!}{2!2!} \times \frac{6!}{2!3!} = 604800$	B1 M1 A1 3	8! ($8 \times 7!$) or 6! seen anywhere, either alone or in numerator) Dividing by at least 3 of 2!2!2!3! (may be fractions added) Correct answer																												
(b)	<table> <tr> <td>C(7)</td><td>E(6)</td><td>A(4)</td><td></td></tr> <tr> <td>1</td><td>1</td><td>2</td><td>$= 7 \times 6 \times {}^4C_2 = 252$</td></tr> <tr> <td>1</td><td>2</td><td>1</td><td>$= 7 \times {}^6C_2 \times 4 = 420$</td></tr> <tr> <td>1</td><td>3</td><td>0</td><td>$= 7 \times {}^6C_3 \times 1 = 140$</td></tr> <tr> <td>2</td><td>1</td><td>1</td><td>$= {}^7C_2 \times 6 \times 4 = 504$</td></tr> <tr> <td>2</td><td>2</td><td>0</td><td>$= {}^7C_2 \times {}^6C_2 \times 1 = 315$</td></tr> <tr> <td>3</td><td>1</td><td>0</td><td>$= {}^7C_3 \times 6 \times 1 = 210$</td></tr> </table> Total = 1841	C(7)	E(6)	A(4)		1	1	2	$= 7 \times 6 \times {}^4C_2 = 252$	1	2	1	$= 7 \times {}^6C_2 \times 4 = 420$	1	3	0	$= 7 \times {}^6C_3 \times 1 = 140$	2	1	1	$= {}^7C_2 \times 6 \times 4 = 504$	2	2	0	$= {}^7C_2 \times {}^6C_2 \times 1 = 315$	3	1	0	$= {}^7C_3 \times 6 \times 1 = 210$	M1 A1 M1* DM1 A1 5	Mult 3 appropriate combinations together assume $6 = {}^6C_1$, $1 = {}^4C_0$ etc., $\sum r=4$, C&E both present At least 3 correct unsimplified products Listing at least 4 different correct options Summing at least 4 outcomes, involving 3 combs or perms, $\sum r=4$ Correct answer SC if CE removed, M1 available for listing at least 4 different correct options for remaining 2. DM1 for ${}^7C_1 \times {}^6C_1 \times (\text{sum of at least 4 outcomes})$
C(7)	E(6)	A(4)																													
1	1	2	$= 7 \times 6 \times {}^4C_2 = 252$																												
1	2	1	$= 7 \times {}^6C_2 \times 4 = 420$																												
1	3	0	$= 7 \times {}^6C_3 \times 1 = 140$																												
2	1	1	$= {}^7C_2 \times 6 \times 4 = 504$																												
2	2	0	$= {}^7C_2 \times {}^6C_2 \times 1 = 315$																												
3	1	0	$= {}^7C_3 \times 6 \times 1 = 210$																												

Q7

(a) (i)	$9 \times 9 \times 8$	M1 M1	Logical listing attempt
	$= 648$	A1 [3]	
	OR $900 - 28 \times 9 = 648$		
(ii)	(7...in $1 \times 8 \times 4 = 32$ ways	M1	Listing #s starting with 7 or 9 and ending odd
	8 ...in $1 \times 8 \times 5 = 40$	M1	
	9... in $1 \times 8 \times 4 = 32$	M1	
	Total 104 ways	A1 [4]	
(b)	R(6) T(5) D(4)		Mult 3 combs, ${}^6C_x \times {}^5C_y \times {}^4C_z$ Summing 2 or 3 three-factor outcomes can be perms, + instead of $\times$ 2 options correct unsimplified
	$2\ 2\ 3 = {}^6C_2 \times {}^5C_2 \times {}^4C_3 = 600$	M1	
	$2\ 3\ 2 = {}^6C_2 \times {}^5C_3 \times {}^4C_2 = 900$	M1	
	$3\ 2\ 2 = {}^6C_3 \times {}^5C_2 \times {}^4C_2 = 1200$	A1	
	Total = 2700	A1 [4]	

Q8

(a) (i)	$\frac{10!}{2!3!} = 302400$	B1 [1]	Exact value only, isw rounding
(ii)	e.g. *W*****W*, **W*****W, W*****W**	M1	8! Seen mult or alone. Cannot be embedded (arrangements of other 8 letters).
	$\frac{8!}{3!} \times 3$ (for the Ws)	M1	
		M1	
	= 20160	A1 [4]	
(b)	S(5) A(7) C(4)	M1	Mult 3 combinations, ${}^5C_x, {}^7C_y, {}^4C_z$ (not $5 \times 7 \times 4$)
	1 3 2 : $5 \times {}^7C_3 \times {}^4C_2 = 1050$		
	1 4 1 : $5 \times {}^7C_4 \times 4 = 700$		
	2 3 1 : ${}^5C_2 \times {}^7C_3 \times 4 = 1400$	A1	
	3 2 1 : ${}^5C_3 \times {}^7C_2 \times 4 = 840$		
	(Outcomes : Options)	M1	
	Total = 3990	A1 [4]	

Q9

(i)	7560 ways	B1	[1]	
(ii)	RxxxxxxG in $\frac{7!}{4!}$	B1		7! alone seen in num or 4! alone in denom Must be in a fraction. $\frac{7! \times 2}{4! \times 2}$ gets full marks
	= 210 ways	B1	[2]	
(iii)	eg EEEEExxxx in $\frac{6!}{2!}$	B1		6! or $5! \times 6$ seen in numerator or on own Can be $6! \times k$ but not $6! \pm k$
	= 360 ways	B1	[2]	
(iv)	1 R eg RVG or RVN or RGN = 3	B1	[1]	
(v)	no Rs eg VGN or 3C3 ways = 1 2 Rs eg RRV or 3C1 ways = 3	M1		Summing at least 2 options for R
	Total = 7	A1		Correct outcome for no Rs or 2 Rs – evaluated
		A1	[3]	

Q10

(a)	e.g. P*N*P*P*L $= \frac{5!}{3!} \times \frac{{}^6P_4}{2!}$ = 3600	M1		Mult by 5! in num
		M1		Dividing by 3! or 2!
		M1		Mult by 6P_4 oe
		A1	[4]	
(b) (i)	${}^7C_5 \times {}^5C_4 \times {}^2C_1 \times {}^2C_1$ = 420	M1		Mult 4 combs of which three are correct
		A1	[2]	
(ii)	both in team ${}^6C_4 \times {}^4C_3 \times 2 \times 2 = 240$ $420 - 240 = 180$ ways OR Bat in bowl out + bowl in bat out + both out $= {}^6C_4 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_4 \times 2 \times 2$ $= 60 + 96 + 24 = 180$ ways OR Bat in bowl out + bat out $= 60 + {}^6C_5 \times {}^5C_4 \times 2 \times 2 = 60 + 120 = 180$ ways	M1		Evaluating both in team and subtracting from (i) 240 seen can be unsimplified ft their 420, their 240
		M1		
		A1		
		M1		summing 2 or 3 options not both in team
		A1		2 or 3 options correct unsimplified
		A1		Correct ans from correct working
		M1		As above, or bowl in bat out + bowl out
		A1 A1	[3]	

Q11

(i)	e.g. (OAE)(CPNHGN) or cv $\frac{4!}{2!} \times \frac{6!}{2!} \times 2 = 8640$	M1 M1 A1	[3]	4!/2! or 6!/2! seen anywhere All multiplied by 2 oe
(ii)	First Method Total ways = $10!/2!2! = 907200$ EE together in $9!/2!$ ways = 181440 EE not together = $907200 - 181440 = 725760$ OR Second Method C P N H G N O A in $8!/2!$ ways ↑ Insert E in 9 ways Insert 2nd E in 8 ways, $\div 2$ Total = $8!/2! \times 9 \times 8 \div 2 = 725760$	B1 M1 M1 A1 B1 M1 M1 A1	[4]	Total ways together correct EE together attempt alone Considering total – EE together 8!/2! Seen Interspersing an E, x n where n=7,8,9. Condone additional factors. Mult by $9 \times 8 (\div 2)$, 9C_2 or 9P_2 only oe
(iii)	First Method EN** in 6C_2 ways = 15 different ways EENN in 1 way Total 16 ways OR Second Method Listing with at least 8 different correct options Listing all correct options Total = 15 different ways EENN in 1 way Total 16 ways	M1 M1 A1 B1 A1 M1 M1 A1 B1 A1	[5]	6C_x or yC_2 seen alone or mult by $k > 1$, $x < 6$, $y > 2$ (1x1x) 6C_2 seen strictly alone or added to their EENN only Value stated or implied by final answer correct value stated Award 16 SRB2 if no method is present

Q12

(i)	e.g. **5 in 3P_2 ways = 6 **7 in ${}^3P_2 = 6$ Total 12 AG	M1 M1 A1	[3]	Recognising ends in 5 or 7, can be implied Summing ends in 5 + ends in 7 oe Correct answer following legit working
	OR listing 457, 547, 467, 647, 567, 657, 475, 745, 465, 645, 675, 765 Total 12 AG	M1 M1 A1		Listing at least 5 different numbers ending in 5 Listing at least 5 different numbers ending in 7
(ii)	1 digit in 2 ways 2 digits in *5 or *7 = ${}^3P_1 \times 2 = 6$ 4 digits in ***5 or ***7 = ${}^3P_3 \times 2 = 12$ Total ways = 32	M1 A1 A1	[3]	Consider at least 3 options with different number of digits. If no working, must be 3 or 4 from 2, 6, 12, 12 One option correct from 1, 2 or 4 digits

Q13

(a)	<i>EITHER:</i> e.g. xxxxx =5! for the other children				(B1)	5! OE seen alone or mult by integer $k \geq 1$, no addition			
	Put y in 6 ways, then 5 then 4 for the youngest children				B1	Mult by 6P3 OE			
	Answer $5! \times 6P3 = 14400$				B1)	Correct answer			
	<i>OR:</i> total – 3 tog – 2 tog = $8! - 6!3! - 6! \times 2 \times 5 \times 3 = 14400$				(B1)	$8! - 6! \times k \geq 1$ seen			
					B1	$6!3!$ or $6! \times 2 \times 5 \times 3$ seen subtracted			
					B1)	Correct answer			
	Total:				3				
(b)	D 2	W 2	M 1	=	$6C2 \times 4C2 \times 1$	=	90	B1	One correct unsimplified option
	3	1	1	=	$6C3 \times 4 \times 1$	=	80	M1	Summing 2 or more 3-factor options which can contain perms or 3 factors added. The 1 can be implied
	1	3	1	=	$6 \times 4C3 \times 1$	=	24	M1	Summing the correct 3 unsimplified outcomes only
	Total=194 ways				A1				
	Total:				4				
(c)	C 2	D 1	S 1	=	${}^{26}C_2 \times 9 \times 5 \times 4!$	=	351 000	M1	summing 2 or more options of the form (2 1 1), (1 2 1), (1 1 2), can have perms, can be added
	1	2	1	=	$26 \times {}^9C_2 \times 5 \times 4!$	=	112 320	M1	4 relevant products seen excluding 4! e.g. $26 \times 9 \times 8 \times 5$ or $26 \times {}^9P_2 \times 5$ for 2nd outcome, condone $26 \times 9 \times 5 \times 37$ as being relevant
	1	1	2	=	$26 \times 9 \times {}^5C_2 \times 4!$	=	56 160	M1	mult all terms by 4! or 4!/2!
	Total = 519 480				A1				
	Total:				4				

Q14

(i)	<i>EITHER: Route 1</i> <i>A</i> ***** <i>A</i> in $9! / 2!2!5! = 756$ ways	(*M1)	Considering <i>AA</i> and <i>BB</i> options with values
	<i>B</i> ***** <i>B</i> in $9! / 4!5! = 126$ ways	A1	Any one option correct
	$756 + 126$	DM1	Summing their <i>AA</i> and <i>BB</i> outcomes only
	Total = 882 ways	A1)	
	<i>OR1: Route 2</i> <i>A</i> ***** <i>A</i> in ${}^9C_5 \times {}^4C_2 = 756$ ways	(M1)	Considering <i>AA</i> and <i>BB</i> options with values
	<i>B</i> ***** <i>B</i> in ${}^9C_4 \times {}^5C_5 = 126$ ways	A1	Any one option correct
	$756 + 126$	DM1	Summing their <i>AA</i> and <i>BB</i> outcomes only
	Total = 882	A1)	
	Total:	4	

(ii)	<i>EITHER:</i> (The subtraction method) <i>As</i> together, no restrictions $8! / 2!5! = 168$	(*M1)	Considering all <i>As</i> together – 8! seen alone or as numerator – condone $\times 4!$ for thinking <i>A</i> 's not identical
	<i>As</i> together and <i>Bs</i> together $7! / 5! = 42$	M1	Considering all <i>As</i> together and all <i>Bs</i> together – 7! seen alone or numerator
		M1	Removing repeated <i>Bs</i> or <i>Cs</i> – Dividing by 5! either expression or 2! 1st expression only – OE
	Total $168 - 42$	DM1	Subt their 42 from their 168 (dependent upon first M being awarded)
	$= 126$	A1)	
	<i>OR1:</i> <i>As</i> together, no restrictions ${}^8C_5 \times {}^3C_1 = 168$	(*M1)	8C_5 seen alone or multiplied
		M1	7C_5 seen alone or multiplied
	<i>As</i> together and <i>Bs</i> together ${}^7C_5 \times {}^2C_1 = 42$	M1	First expression $\times {}^3C_1$ or second expression $\times {}^2C_1$
	Total $168 - 42$	DM1	Subt their 42 from their 168 (dependent upon first M being awarded)
	$= 126$	A1)	
	<i>OR2:</i> (The intersperse method)	(M1)	Considering all “ <i>As</i> together” with <i>Cs</i> – Mult by 6!
	(<i>AAAA</i>) <i>CCCCC</i> then intersperse <i>B</i> and another <i>B</i>	M1	Removing repeated <i>Cs</i> – Dividing by 5!– [Mult by 6 implies M2]
		*M1	Considering positions for <i>Bs</i> – Mult by 7P2 oe –
	$\frac{6!}{5!} \times 7 \times 6 \div 2$	DM1	Dividing by 2! Oe – removing repeated <i>Bs</i> (dependent upon 3rd M being awarded)
	$= 126$	A1)	
	Total:	5	

Q15

(a)(i)	First digit in 2 ways. $2 \times 4 \times 3 \times 2$ or $2 \times 4P3$	M1	1, 2 or $3 \times 4P3$ OE as final answer
	Total = 48 ways	A1	
	Totals:	2	
(a)(ii)	$2 \times 5 \times 5 \times 3$	M1	Seeing 5^2 mult; this mark is for correctly considering the middle two digits with replacement
	= 150 ways	M1	Mult by 6; this mark is for correctly considering the first and last digits
	Totals:	A1	
(b)(i)	OO**** in $^{18}C_4$ ways	M1	$^{18}C_x$ or the sum of five 2-factor products with $n = 14$ and 4, may be $\times$ by $2C2$: $4C0 \times 14C4 + 4C1 \times 14C3 + 4C2 \times 14C2 + 4C3 \times 14C1 + 4C4 (\times 14C0)$
	= 3060	M1	
	Totals:	A1	
(b)(ii)	Choc Not Choc 0 $6 = 1 \times ^{16}C_6 = 8008$ 0.2066 1 $5 = ^4C_1 \times ^{16}C_5 = 17472$ 0.4508 2 $4 = ^4C_2 \times ^{16}C_4 = 10920$ 0.2817 OR Choc Oats Ginger 0 0 6 0 1 5 0 2 4 1 0 5 1 1 4 1 2 3 2 0 4 2 1 3 2 2 2	B1	The correct number of ways with one of 0, 1 or 2 chocs , unsimplified or any three correct number of ways of combining choc/oat/ginger, unsimplified
	Total = 36400 ways	B1	sum the number of ways with 0, 1 and 2 chocs and two must be totally correct, unsimplified OR sum the nine combinations of choc, ginger, oats, six must be totally correct, unsimplified
	Probability = $36400 / ^{20}C_6$	M1	dividing by $^{20}C_6$ (38760) oe
	= 0.939 (910/969)	M1	
	Totals:	A1	
		4	

Q16

(a)(i)	${}^{40}P_5$	M1	${}^{40}P_x$ or 3P_5 oe seen, can be mult by $k \geq 1$
	$= 78\,960\,960$	A1	
		2	
(a)(ii)	not front row e.g. WEJ** in $3 \times 3! = 18$ ways	B1	$3!$ seen mult by $k \geq 1$
	7 rows in $7 \times 18 = 126$ ways	B1	mult by 7
	front row: e.g. *MA** in $4 \times 2 = 8$ ways	M1	attempt at front row arrangements and multiplying by the 7 other rows arrangements, need not be correct
	Total $126 \times 8 = 1008$	A1	
		4	
(b)	<i>EITHER:</i> e.g. *R** in 8C_3 ways = 56 ways *L** in ${}^8C_3 = 56$ ways	(M1)	Considering either R or L only in team
	**** in ${}^8C_4 = 70$ ways	M1*	Considering neither in team
		DM1	summing 3 scenarios
	Total 182 ways	A1)	
	<i>OR1:</i> No restrictions ${}^{10}C_4 = 210$ ways	(M1)	${}^{10}C_4 -$, Considering no restrictions with subtraction
	RL = ${}^8C_2 = 28$	M1*	Considering both in team
	$210 - 28$	DM1	subt
	$= 182$ ways	A1)	
	<i>OR2:</i> R out in ${}^9C_4 = 126$ ways L out in ${}^9C_4 = 126$ ways	(M1)	Considering either R out or L out
	Both out in ${}^8C_4 = 70$	M1*	Considering both out
(b)		DM1	Summing 2 scenarios and subtracting 1 scenario
	$126 + 126 - 70 = 182$ ways.	A1)	
		4	

Q17

(a)(i)	<i>EITHER:</i> 3**, 4**, 6**, 8**	(M1)	5P_2 or ${}^5C_2 \times 2!$ or 5×4 OE (considering final 2 digits)
	options $4 \times 5 \times 4 = 80$	M1	Mult by 4 or summing 4 options (considering first digit)
		A1)	Correct final answer
	<i>OR:</i> Total number of values: $6 \times 5 \times 4 = 120$	(M1)	Calculating total number of values (with subtraction seen)
	Number of values less than 300: $2 \times 5 \times 4 = 40$	M1	Calculating number of unwanted values
	Number of evens = $120 - 40 = 80$	A1)	Correct final answer
(a)(ii)	3**, 4**, 6**, 8** <i>EITHER:</i> options $4 \times 6 \times 4$ (last)	3	
		(M1)	6 linked to considering middle digit e.g. multiplied or in list
		M1	Multiply an integer by 4×4 (condone $\times 16$) (No additional figures present for both M's to be awarded)
		A1)	
	= 96		
	<i>OR:</i> Total number of values $4 \times 6 \times 6 = 144$	(M1)	Calculating total number of values (with subtraction seen)
	Number of odd values $4 \times 6 \times 2 = 48$	M1	Calculating number of unwanted values
	Number of evens = $144 - 48 = 96$	A1)	
		3	
(b)(i)	252	B1	
(b)(i)	252	1	
		B1	
(b)(ii)	B (6)G(4) 5 0 in ${}^6C_5 (\times {}^4C_0) = 6 \times 1 = 6$ 4 1 in ${}^6C_4 \times {}^4C_1 = 15 \times 4 = 60$ 3 2 in ${}^6C_3 \times {}^4C_2 = 20 \times 6 = 120$	1	
		M1	Multiplying 2 combinations ${}^6C_q \times {}^4C_r$, $q + r = 5$, or 6C_5 seen alone
		M1	Summing 2 or 3 appropriate outcomes, involving perm/comb, no extra outcomes.
	Total = 186 ways	A1	
		3	

Q18

(i)	${}^{18}P_5$	M1	${}^{18}P_x$ or yP_5 OE seen, $0 < x < 18$ and $5 < y < 18$, can be mult by $k \geq 1$
	= 1 028 160	A1	
(ii)	<i>EITHER:</i> e.g. *** (CCCC)***** in $5! \times 14$ ways	2 (B1)	$5!$ OE mult by $k \geq 1$, considering the arrangements of cars next to each other
	= 1680	B1	Mult by 14 OE, (or 14 on its own) considering positions within the line
	P (next to each other) = $1680/1\,028\,160$	M1	Dividing by (i) for probability
	P(not next to each other) = $1 - 1680/1\,028\,160$	M1	Subtracting prob from 1 (or their ' $5! \times 14$ ' from (i))
	$= 0.998 \left(\frac{611}{612} \right)$ OE	A1)	
	<i>OR1:</i> $\frac{5! \times 14!}{18!} = 0.001634$	(B1)	$5!$ OE mult by $k \geq 1$ (on its own or in numerator of fraction) considering the arrangements of cars next to each other
		B1	Multiply by $14!$, (or $14!$ on its own) considering all ways of arranging spaces with 5 cars together
		M1	Dividing by $18!$, total number of ways of arranging spaces
	$1 - 0.001634$	M1	Subtracting prob from 1 (or ' $5! \times 14!$ ' from $18!$)
	= 0.998(366)	A1)	
	<i>OR2:</i> 4 together – $2 \times 5! \times 14C12 = 21\,840$ 3, 1, 1 – $3 \times 5! \times 14C11 = 131\,040$ 3, 2 – $2 \times 5! \times 14C12 = 21\,840$ 2,2,1 – $3 \times 5! \times 14C11 = 131\,040$ 2,1,1,1 – $4 \times 5! \times 14C10 = 480\,480$ 1,1,1,1,1 – $5! \times 14C9$ or $14P5 = 240\,240$	(M1)	Listing the six correct scenarios (only): 4 together; 3 together and 2 separate; 3 together and 2 together; two sets of 2 together and 1 separate; 2 together and 3 separate; 5 separate.
	Total = 1 026 480	M1 A1	Summing total of the six scenarios, at least 2 correct unsimplified Total of 1 026 480
		M1	Dividing their 1 026 480 by their 6(i)
	$1\,026\,480 \div 1\,028\,160 = 0.998(366)$	A1)	
		5	
(iii)	R(5) W(4) B(3) Scenarios No. of ways 1 1 1 = $5 \times 4 \times 3 = 60$ 0 1 2 = $4 \times {}^3C_2 = 12$ 0 2 1 = ${}^4C_2 \times 3 = 18$ 1 0 2 = $5 \times {}^3C_2 = 15$ 2 0 1 = ${}^5C_2 \times 3 = 30$ 1 2 0 = $5 \times {}^4C_2 = 30$ 2 1 0 = ${}^5C_2 \times 4 = 40$	B1	$5C1 \times 4C1 \times 3C1$ or better seen i.e. no. of ways with 3 different colours
		M1	Any of 5C_2 or 4C_2 or 3C_2 seen multiplied by $k > 1$ (can be implied)
		A1	2 correct unsimplified 'no. of ways' other than $5C1 \times 4C1 \times 3C1$
		M1	Summing no more than 7 scenario totals containing at least 6 correct scenarios
	Total = 205	A1	
	OR		
	${}^{12}C_3 -$	M1	Seeing ' ${}^{12}C_3 -$ ', considering all selections of 3 cars
	$- {}^5C_3$	M1	Subt 5C_3 OE, removing only red selections
	$- {}^4C_3$	M1	Subt 4C_3 OE, removing only white selections
	$- {}^3C_3$	M1	Subt 3C_3 OE, removing only black selections
	= 205	A1	Correct answer
		5	

Q19

(i)	$\frac{9!}{2!2!} = 90720$	B1	Must see 90720
		1	
(ii)	Method 1 ↑ * * * * * A	B1	5! seen multiplied (arrangement of consonants allowing repeats)
	No. arrangements of consonants $\times$ ways of inserting vowels =	B1	6P_4 oe (i.e. $6 \times 5 \times 4 \times 3$, ${}^6C_4 \times 4!$) seen mult (allowing repeats) no extra terms
	$\frac{5!}{2!}$ $\times \frac{{}^6P_4}{2!}$	B1	Dividing by at least one 2! (removing at least one set of repeats)
	Answer $\frac{{}^6P_4}{2!} \times \frac{5}{2} = 10\,800$	B1	Correct final answer
		4	
(iii)	${}^5C_3 = 10$	M1	5C_x or 5P_x seen alone, $x = 2$ or 3
		A1	Correct final answer not from 5C_2
		2	
(iv)	Method 1 Considering separate groups	M1	Considering two scenarios of MME or EEM or MMEE with attempt, may be probs or perms
	MME** = ${}^5C_2 = 10$ MEE** = ${}^5C_2 = 10$ MMEE* = ${}^5C_1 = 5$	M1	Summing three appropriate scenarios from the four need 5C_x seen in all of them
	ME*** = ${}^5C_3 = 10$ see (iii) Total = 35	A1	Correct final answer
	Method 2 Considering criteria are met if ME are chosen	M1	7C_x only seen, no other terms
		M1	5C_3 only seen, no other terms
	ME *** = ${}^7C_3 = 35$	A1	Correct final answer
		3	

Q20

(a)(i)	(AAAIU) * * * * Arrangements of vowels/repeats × arrangements of (consonants & vowel group) =	M1	$k \times 5!$ (k is an integer, $k \geq 1$)
	$\frac{5! \times 5!}{3!}$	M1	$\frac{m}{3}!$ (m is an integer, $m \geq 1$) Both Ms can only be awarded if expression is fully correct
	= 2400	A1	Correct answer
		3	
(a)(ii)	E.g. R * * * T * * * L . Arrangements of consonants RL, RS, SL = ${}^3P_2 = 6$ Arrangements of remaining letters = $\frac{6!}{3!} = 120$	M1	$k \times \frac{6!}{3!}$ or $k \times {}^3P_2$ or $k \times {}^3C_2$ or $k \times 3!$ or $k \times 3 \times 2$ (k is an integer, $k \geq 1$), no irrelevant addition
	Total 120×6	M1	Correct unsimplified expression or $\frac{6!}{3!} \times {}^3C_2$
	= 720 ways	A1	Correct answer
		3	
(b)	Method 1 N(2) R(8) Br(4) 1 2 1 = $2 \times {}^8C_2 \times 4 = 224$	M1	Multiply 3 combinations, ${}^2C_x \times {}^8C_y \times {}^4C_z$. Accept ${}^2C_1 = 2$ etc.
	2 1 1 = $1 \times {}^8C_1 \times 4 = 32$ 1 1 2 = $2 \times 8 \times {}^4C_2 = 96$	A1	3 or more options correct unsimplified
	2 0 2 = $1 \times 1 \times {}^4C_2 = 6$ 1 0 3 = $2 \times 1 \times 4 = 8$	M1	Summing <i>their</i> values of 4 or 5 legitimate scenarios (no extra scenarios)
	Total = 366 ways	A1	Correct answer
	Method 2 ${}^{14}C_4 - (2N2R \text{ or } 1N3R \text{ or } 4R \text{ or } 3R1B \text{ or } 2R2B \text{ or } 1R3B \text{ or } 4B)$	M1	' ${}^{14}C_4 - k$ ' seen, k an integer from an expression containing 8C_x
	$1001 - (1 \times {}^8C_2 + 2 \times {}^8C_3 + {}^8C_4 + {}^8C_3 \times 4 + {}^8C_2 \times {}^4C_2 + 8 \times 4 + 1)$	A1	4 or more 'subtraction' options correct unsimplified, may be in a list
	$1001 - (28 + 112 + 70 + 224 + 168 + 32 + 1)$	M1	<i>Their</i> ${}^{14}C_4 - [\text{their values of 6 or more legitimate scenarios}]$ (no extra scenarios, condone omission of final bracket)
	= 366	A1	Correct answer
		4	

Q21

(i)	****E****	M1	Mult by 8! or 8P_8 oe (arrangements ignoring repeats)
	Other letters arranged in $\frac{8!}{2!3!}$	A1	Correct final answer www
	= 3360 ways		
	OR $\frac{8 \times 7 \times 6 \times 5 \times 4 \times 4 \times 3 \times 2 \times 1}{4!2!} = 3360$ ways	M1	Correct numerator (161 280)
		A1	Correct final answer www
	Total:	2	
(ii)	* * * * *	M1	k mult by 6C_4 or 6P_4 oe (ways to insert Es ignoring repeats), k can = 1
	↑ Arrangements other letters × ways Es inserted		or k mult by $\frac{5!}{2!}$
	$= \frac{5!}{2!} \times {}^6C_4 \left(\frac{5!}{2!} \times \frac{{}^6P_4}{4!} \right)$	M1	Correct unsimplified expression or $\frac{5!}{2!} \times {}^6P_4$
	= 900 ways	A1	Correct answer
	OR Total no of ways – no of ways with Es touching $9!/(4! \times 2!) - \dots$ or $7\,560 - \dots$ $\frac{6!}{2!} + {}^6P_2 \times \frac{5!}{2!} + \frac{{}^6P_2 \times 5!}{2! \times 2!} + \frac{{}^6P_3}{2! \times \frac{5!}{2!}}$ $= 360 + 1800 + 900 + 3600 = 6660$	M1	7560 unsimplified – k
(ii)	$7\,560 - 6\,660 = 900$	A1	Correct answer
	OR Adding the number of ways with the first E in the 1 st (E ₁), 2 nd (E ₂) or 3 rd (E ₃) position. $\frac{5!}{2!} (E_1 + E_2 + E_3)$ where $E_1 = 10, E_2 = 4, E_3 = 1$	M1	For any values for E ₁ , E ₂ and E ₃
	$\frac{5!}{2!} (E_1 + E_2 + E_3)$	M1	For any two correct values of E ₁ , E ₂ and E ₃
	$600 + 240 + 60 = 900$	A1	Correct answer
	Total:	3	
(iii)	EENN* in 3 ways	B1	Numerical value must be stated
	Total:	1	
(iv)	EE *** with no N: 1 way EEN** 3C2 or listing 3 ways EENN* 3 ways from (iii)	M1	Identifying the three different scenarios of EE, EEE or EEEE
		A1	Total no of ways with two Es (7 or 3 + 3 + 1)
	EEE** with no N: 3 ways EEEN* 3 ways EEENN 1 way	A1	Total no. of ways with 3 Es (7)
	EEEE* no N 3 ways EEEEN 1 way Total 18 ways	A1	Correct answer stated
	Method List containing ways with 2Es, 3Es and 4Es List containing at least 8 correct different ways List of all 18 correct ways Total 18	M1	At least 1 option listed for each of EE^^, EEE^^, EEEEE^
		A1	Ignore repeated options
		A1	Ignore repeated/incorrect options
		A1	Correct answer stated
	Total:	4	

Q22

(i)	Scenarios are: 4V + 1C + 1DB: ${}^{11}C_4 \times {}^5C_1 \times {}^4C_1$	M1	${}^{11}C_a \times {}^5C_b \times {}^4C_c, a+b+c=6,$
	4V + 2C: ${}^{11}C_4 \times {}^5C_2$ 5V + 1C: ${}^{11}C_5 \times {}^5C_1$	B1	2 correct unsimplified options
	6600 + 3300 + 2310	M1	Add 2 or 3 correct scenarios only
	= 12210	A1	Correct answer
		4	
(ii)	$4! \times 3!$	M1	k multiplied by 3! or 4!, k an integer ≥ 1
		A1	Correct unsimplified expression
	= 144	A1	Correct answer
		3	

Q23

(i)	$\frac{11!}{4!4!2!}$	M1	$\frac{11!}{4!k} \text{ or } \frac{11!}{2!k}, k \text{ a positive integer}$
	= 34650	A1	Correct final answer
		2	
(ii)	Method 1		
	$P(SS) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110} (= 0.10911)$	B1	One of P(SS), P(PP) or P(II) correct, allow unsimplified
	$P(PP) = \frac{2}{11} \times \frac{1}{10} = \frac{2}{110} (= 0.01818)$ $P(II) = \frac{4}{11} \times \frac{3}{10} = \frac{12}{110} (= 0.10911) \frac{4}{11} \times \frac{3}{10}$	M1	Sum of probabilities from 3 appropriate identifiable scenarios (either by labelling or of form $\frac{4}{11} \times \frac{a}{b} + \frac{2}{11} \times \frac{c}{b} + \frac{4}{11} \times \frac{a}{b}$ where $a = 4 \text{ or } 3, b = 11 \text{ or } 10, c = 2 \text{ or } 1$)
	Total = $\frac{26}{110} = \frac{13}{55}$ oe (0.236)	A1	Correct final answer
	Method 2		
	Total number of selections = ${}^{11}C_2 = 55$ Selections with 2 Ps = 1	B1	Seen as the denominator of fraction (no extra terms) allow unsimplified
	Selections with 2 Ss = ${}^4C_2 = 6$ Selections with 2 Is = ${}^4C_2 = 6,$	M1	Sum of 3 appropriate identifiable scenarios (either by labelling or values, condone use of permutations. May be implied by 2,12,12)
	Total selections with 2 letters the same = 13 Probability of 2 letters the same = $\frac{13}{55}$ oe (0.236)	A1	Correct final answer, without use of permutations
		3	

(24)

(i) $C_1 C_2 C_3 C_4 C_5 C_6 B_1 B_2 B_3$

$$\frac{9 \times 8 \times 7 \times 6}{4!} = {}^9C_4$$

$$\boxed{{}^9C_4 = 126 \text{ ways}}$$

(ii) Fav Bus $\frac{1}{2}$ Car : $B^* C^*$

$$\underbrace{B^*}_{\text{Fixed.}} \underbrace{C^*}_{\text{Fixed.}} \frac{7 \times 6}{2!} = {}^7C_2$$

$$\boxed{{}^7C_2 = 21 \text{ ways}}$$

(iii)

$$\boxed{B_1 B_2 B_3} \quad C_1 C_2 C_3 C_4 C_5 C_6$$

1 2 3 4 5 6 7

Stay together in a block $\therefore$ 7 units here

$$\text{Arrange} = 7! \times 3!$$

ways to arrange the 3 Buses inside block.

$$\therefore \boxed{7! \times 3! = 30,240 \text{ ways}}$$

$$(iv) C_1 \overset{5}{-} C_2 \overset{4}{-} C_3 \overset{3}{-} C_4 - C_5 - C_6$$

$B_1 B_2 B_3 \rightarrow$ How many ways can
Buses go in the 5
slots?

$$5 \times 4 \times 3 \text{ ways } ({}^5P_3)$$

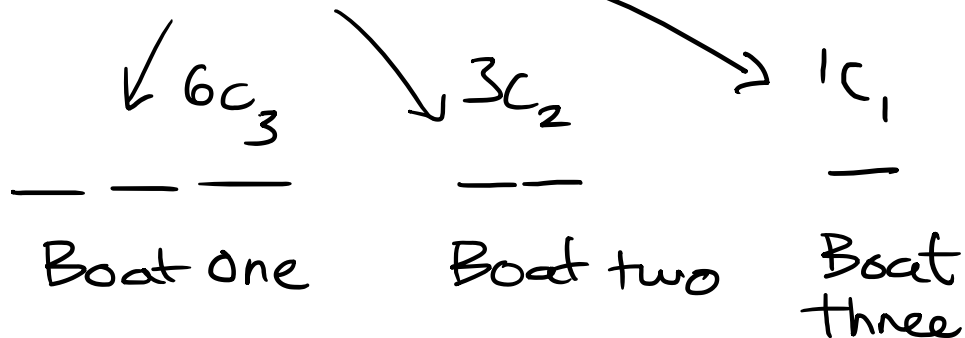
Arrange C_1 to C_6 is : $6!$

$$\therefore \boxed{6! \times 5 \times 4 \times 3 = 43,200 \text{ ways}}$$

(25)

6 teens

(a)



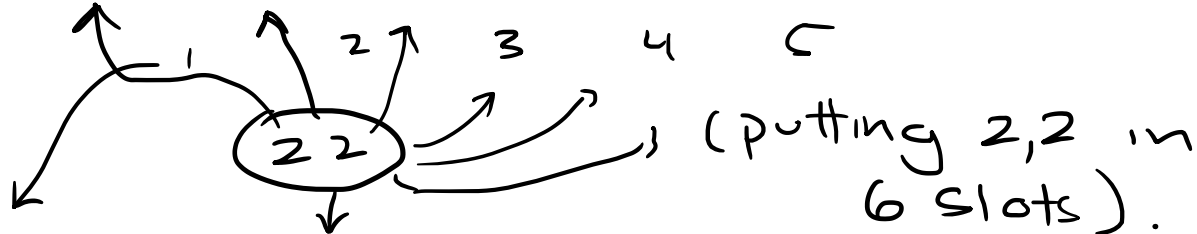
$${}^6C_3 \times {}^3C_2 \times {}^1C_1 = 60 \text{ ways}$$

(b) 2 2 3 7 7 7 8

(i) $\boxed{228} \mid \boxed{3777}$

$$2 \times \left(\frac{3!}{2!} \right) \times \left(\frac{4!}{3!} \right) = 24 \text{ ways}$$

(ii) $_ 3 _ 7 _ 7 _ 7 _ 8 _$



$$\left(\frac{5!}{3!} \right) \times \left(\frac{6 \times 5}{2!} \right) = 300 \text{ ways.}$$

(26) (i)

$$\frac{K}{1} \text{ --- } \frac{u}{2}$$

$9!$

$$2 \times 9! = 725,760 \text{ ways}$$

(ii)

1	2	3	4	5	6
$K_1 K_2 K_3 K_4 K_5$	$u_1 u_2$				
$5!$	$2!$	$\underbrace{\hspace{150px}}_{\text{ADULTS}}$			

$$6! \times 5! \times 2! = 172,800 \text{ ways.}$$

(27) STEEPLECHASE

(i) $EEEE$ STPLCHAS

$$\frac{9!}{2!} = 181,440 \text{ ways}$$

(ii) _ T _ E _ E _ P _ L _ E _ C _ H _ A _ S _ E _

$$\left(\frac{10!}{4!} \right) \times \left(\frac{11 \times 10}{2} \right) = 8,316,000 \text{ ways}$$

$\uparrow$
letters
putting 2 S's
in slots

(ii) TEEPLECHAE

TPLECHAE (6)

(Have to take into account repeated E's)

0 E's : $\underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \frac{6 \times 5 \times 4}{3!} = 20 \text{ ways}$

1 E : $\underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \frac{6 \times 5}{2!} = 15 \text{ ways}$

2 E's : $\underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} 6 \text{ ways}$

3 E's : $\underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad} 1 \text{ way}$

$\therefore 20 + 15 + 6 + 1 = 42 \text{ ways}$

(28) TOADSTOOL

(i) $\boxed{\text{O O O}} \boxed{\text{T T}} \underline{\quad} \underline{\quad} \underline{\quad} \underline{\quad}$
1 2 3 4 5 6

$6! = 720 \text{ ways}$

(ii) $\underline{\quad} \text{O} \underline{\quad} \text{A} \underline{\quad} \text{D} \underline{\quad} \text{S} \underline{\quad} \text{O} \underline{\quad} \text{O} \underline{\quad} \text{L} \underline{\quad}$

$\left(\frac{7!}{3!} \right) \left(\frac{8 \times 7}{2} \right) = 23,520 \text{ ways}$

(iii) $T \text{---} 1 \text{---} 2 \text{---} 3 \text{---} 4 \text{---} 5 \text{---} 6 \text{---} 7 \text{---} T$

(T@Both ends) $\frac{7!}{3!} = 840 \text{ ways}$

$$\text{TOTAL \# ways} = \frac{9!}{3!2!} = 30,240 \text{ ways}$$

$$\text{Prob} = \frac{840}{30,240} = 0.0278$$

(iv) TOADSTOOL

ADSLTO

OOT — $6 \times 5/2$ 15 wgs

29 (i) CORRIDORS

$\overbrace{RRRR}^{3!} \overbrace{OO}^{2!} C I D S$

$$\frac{9!}{3! \times 2!} = 30,240 \text{ ways}$$

(ii) CORRIDORS

D _ _ _ _ _ R

$$\frac{7!}{2!2!} = 1260 \text{ ways}$$

D _ _ _ _ _ O

$$\frac{7!}{3!} = 840 \text{ ways}$$

$$\text{TOTAL} = 1260 + 840 = 2100 \text{ ways}$$

(30) 6A, 5D, 4M

$$(i) \quad \underline{A} \underline{A} \underline{A} \underline{D} \underline{D} \underline{M} \underline{A} \quad \left(\frac{6 \times 5 \times 4 \times 3}{4!} \right) \left(\frac{5 \times 4}{2!} \right) (4) \\ = 600 \text{ ways}$$

$$\underline{A} \underline{A} \underline{A} \underline{D} \underline{D} \underline{M} \underline{D} \quad \left(\frac{6 \times 5 \times 4}{3!} \right) \left(\frac{5 \times 4 \times 3}{3!} \right) (4) \\ = 800 \text{ ways}$$

$$\underline{A} \underline{A} \underline{A} \underline{D} \underline{D} \underline{M} \underline{M} \quad \left(\frac{6 \times 5 \times 4}{3!} \right) \left(\frac{5 \times 4}{2!} \right) \left(\frac{4 \times 3}{2!} \right) \\ = 1200 \text{ ways}$$

$$\boxed{\text{TOTAL} = 2600 \text{ ways}}$$

(ii)

$$\frac{7 \times 6 \times 5 \times 4}{7 \times 6 \times 5 \times 4} \\ 4!$$

$$\frac{3 \times 2 \times 1}{3 \times 2 \times 1} \\ 3!$$

$$\# \text{ of ways} = \left(\frac{7 \times 6 \times 5 \times 4}{4!} \right) \times (1) = 35 \text{ ways.}$$